

NOTES: Section 3.3 – Complete the Square

Goals: #1 – I can solve quadratic equations using square roots.

#2 – I can solve quadratic equations by completing the square.

#3 – I can write quadratic functions in vertex form.

Homework: Section 3.3 Worksheet

EXPLORATION #1: Perfect Square Trinomials

Factor the following expressions:

1.) $x^2 - 6x + 9$

$$\boxed{(x-3)^2}$$

2.) $x^2 + 22x + 121$

$$\boxed{(x+11)^2}$$

What value of c would make the following a perfect square trinomial?

3.) $x^2 + 14x + c$

$$\boxed{49}$$

NOTES:

• Perfect Square Trinomials

$$a^2 + 2ab + b^2$$

Examples:
$$\begin{array}{ccccccc} x^2 & + & 22x & + & 121 & \rightarrow & (x+11)^2 \\ (x)^2 & & 2(11)(x) & & (11)^2 & & \end{array}$$

$$\begin{array}{ccccccc} x^2 & - & 6x & + & 9 & \rightarrow & (x-3)^2 \\ (x)^2 & & 2(3)(x) & & (3)^2 & & \end{array}$$

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EXAMPLE #1: Solving a Quadratic Equation Using Square Roots

Solve each equation by using square roots.

1.) $x^2 - 16x + 64 = 100$

$$(x-8)^2 = 100$$

$$\sqrt{(x-8)^2} = \pm \sqrt{100}$$

$$x-8 = \pm 10$$

$$x = 8 \pm 10$$

$$\boxed{x = 18, -2}$$

2.) $x^2 + 8x + 16 = 45$

$$(x+4)^2 = 45$$

$$\sqrt{(x+4)^2} = \pm \sqrt{45}$$

$$x+4 = \pm 3\sqrt{5}$$

$$\boxed{x = -4 \pm 3\sqrt{5}}$$

MONITORING PROGRESS: On Your Own

Solve each equation by using square roots.

1.) $x^2 + 18x + 81 = 5$

$$(x+9)^2 = 5$$

$$\sqrt{(x+9)^2} = \pm \sqrt{5}$$

$$x+9 = \pm \sqrt{5}$$

$$\boxed{x = -9 \pm \sqrt{5}}$$

2.) $x^2 - 24x + 144 = -100$

$$(x-12)^2 = -100$$

$$\sqrt{(x-12)^2} = \pm \sqrt{-100}$$

$$x-12 = \pm 10i$$

$$\boxed{x = 12 \pm 10i}$$

NOTES:

To complete the square, we adjust the left side of a quadratic equation so that it becomes a perfect square trinomial.

$$\underline{1}x^2 + bx + c \quad c = \left(\frac{b}{2}\right)^2$$

$$\left(x + \frac{b}{2}\right)^2$$

EXAMPLE #2: Making a Perfect Square Trinomial

Find the values of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

1.) $x^2 + 14x + c$

$$c = \left(\frac{14}{2}\right)^2 = (7)^2 = \boxed{49}$$

$$\boxed{(x+7)^2}$$

2.) $x^2 - 2x + c$

$$c = \left(\frac{-2}{2}\right)^2 = (-1)^2 = \boxed{1}$$

$$\boxed{(x-1)^2}$$

MONITORING PROGRESS: On Your Own

Find the values of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

1.) $x^2 + 8x + c$

$$c = \left(\frac{8}{2}\right)^2 = (4)^2 = \boxed{16}$$

$$\boxed{(x + 4)^2}$$

2.) $x^2 - 9x + c$

$$c = \left(-\frac{9}{2}\right)^2 = \boxed{\frac{81}{4}}$$

$$\boxed{\left(x - \frac{9}{2}\right)^2}$$

EXAMPLE #3: Solving by Completing the Square

Solve the equation by completing the square.

1.) $x^2 - 10x + 7 = 0$ $\left(-\frac{-10}{2}\right)^2 = (-5)^2 = 25$

$$x^2 - 10x + \boxed{25} = -7 + \boxed{25}$$

$$(x - 5)^2 = 18$$

$$\sqrt{(x-5)^2} = \pm \sqrt{18}$$

$$\sqrt{9} \sqrt{2}$$

$$x - 5 = \pm 3\sqrt{2}$$

$$\boxed{x = 5 \pm 3\sqrt{2}}$$

$$\left(\frac{8}{2}\right)^2 = (4)^2 = 16$$

2.) $x^2 + 8x = -20$

$$x^2 + 8x + \boxed{16} = -20 + \boxed{16}$$

$$(x + 4)^2 = -4$$

$$\sqrt{(x+4)^2} = \pm \sqrt{-4}$$

$$x + 4 = \pm 2i$$

$$\boxed{x = -4 \pm 2i}$$

MONITORING PROGRESS: On Your Own

Solve the equation by completing the square.

1.) $x^2 - 18x + 5 = 0$ $\left(-\frac{-18}{2}\right)^2 = (-9)^2 = 81$

$$x^2 - 18x + \boxed{81} = -5 + \boxed{81}$$

$$(x - 9)^2 = 76$$

$$\sqrt{(x-9)^2} = \pm \sqrt{76}$$

$$\sqrt{4} \sqrt{19}$$

$$x - 9 = \pm 2\sqrt{19}$$

$$\boxed{x = 9 \pm 2\sqrt{19}}$$

$$\left(\frac{14}{2}\right)^2 = (7)^2 = 49$$

2.) $x^2 + 14x + 8 = 0$

$$x^2 + 14x + \boxed{49} = -8 + \boxed{49}$$

$$(x + 7)^2 = 41$$

$$\sqrt{(x+7)^2} = \pm \sqrt{41}$$

$$x + 7 = \pm \sqrt{41}$$

$$\boxed{x = -7 \pm \sqrt{41}}$$

EXAMPLE #4: Solving by Completing the Square

Solve the equation by completing the square.

1.) $\frac{3x^2 + 12x - 18}{3} = \frac{0}{3} \quad \left(\frac{4}{2}\right)^2 = (2)^2 = 4$

$$x^2 + 4x + \boxed{4} = 6 + \boxed{4}$$

$$(x+2)^2 = 10$$

$$\sqrt{(x+2)^2} = \pm \sqrt{10}$$

$$x+2 = \pm \sqrt{10}$$

$$\boxed{x = -2 \pm \sqrt{10}}$$

2.) $\frac{2x^2 + 8x + 14}{2} = \frac{0}{2}$

$$x^2 + 4x + \boxed{4} = -7 + \boxed{4}$$

$$(x+2)^2 = -3$$

$$\sqrt{(x+2)^2} = \pm \sqrt{-3}$$

$$x+2 = \pm i\sqrt{3}$$

$$\boxed{x = -2 \pm i\sqrt{3}}$$

EXAMPLE #5: Writing a Quadratic Function in Vertex Form

Write the quadratic function in vertex form. Then identify the vertex.

1.) $y = x^2 - 12x + 18 \quad \left(\frac{-12}{2}\right)^2 = (-6)^2 = 36$

$$\boxed{36} + y = x^2 - 12x + \boxed{36} + 18$$

$$36 + y = (x-6)^2 + 18$$

$$-36 \quad -36$$

$$\boxed{y = (x-6)^2 - 18}$$

$$\boxed{\text{Vertex: } (6, -18)}$$

2.) $y = x^2 + 6x + 4 \quad \left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$$\boxed{9} + y = x^2 + 6x + \boxed{9} + 4$$

$$9 + y = (x+3)^2 + 4$$

$$-9 \quad -9$$

$$\boxed{y = (x+3)^2 - 5}$$

$$\boxed{\text{Vertex: } (-3, -5)}$$

MONITORING PROGRESS: On Your Own

Write the quadratic function in vertex form. Then identify the vertex.

1.) $y = x^2 - 8x + 18 \quad \left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

$$\boxed{16} + y = x^2 - 8x + \boxed{16} + 18$$

$$16 + y = (x-4)^2 + 18$$

$$-16 \quad -16$$

$$\boxed{y = (x-4)^2 + 2}$$

$$\boxed{\text{Vertex: } (4, 2)}$$